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SOME TOPOLOGY PROPERTIES OF $\delta\mathcal{F}_p^m$ CLASS

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Abstract: *In this paper, we will present a subset of delta m-subharmonic functions,* $\delta\mathcal{F}_p^m(\Omega), p \geq 0$. *We then equip one topology for this space and prove that the vector space* $\delta\mathcal{F}_p^m$ *is a locally convex topological space and a Frechet space as well.*

Keywords: δ *-plurisubharmonic functions, m-subharmonic functions, m-Hessian operator, Monge-Ampère operator, weighted functions, locally convex topological vector space.*

1. Introduction

The complex Monge-Ampère operator $(dd^c.)^n$ is well defined over the class of locally bounded plurisubharmonic functions (see [1]). Recently, to extend the domain of definition of this operator for plurisubharmonic functions which are not neccessary to be locally bounded, in [6], [7] Cegrell introduced and investigated the classes $\mathcal{E}_0(\Omega), \mathcal{F}_p(\Omega), \mathcal{E}_p(\Omega), \mathcal{F}(\Omega), \mathcal{E}(\Omega)$ on which the complex Monge-Ampère operator is well defined. He has developed pluripotential theory on these classes. To extend the class of plurisubharmonic functions and to study a class of the complex differential operators more general than the Monge-Ampère operator, in [2] and [10], the authors introduced *m*-subharmonic functions and studied the complex Hessian operator. They also were interested in the complex Hessian equations in $\mathbb{C}^n$ and on compact Kahler manifolds. Continuing to study the complex Hessian operator for *m*-subharmonic functions which may be not locally bounded, in recent preprint, Lu Hoang Chinh introduced the Cegrell classes $\mathcal{E}_m^0(\Omega), \mathcal{F}_m(\Omega)$ and $\mathcal{E}_m(\Omega)$ associated to *m*-subharmonic functions (for details, see [9]) and has proved the complex Hessian operator is well defined on these classes.

On the other hand, convex, subharmonic and plurisubharmonic functions are all convex cones in some larger linear space. Given any such cone, K say, we can investigate the space of differences from this cone δK. Such studies are often motivated by algebraic completion of the cone, and differences of convex functions were considered by F. Riesz in as early as 1911. The δ - convex functions, or d.c. functions as they sometimes are denoted, were studied by Kiselman [13], and Cegrell [5], and have been given attention in many areas ranging from nonsmooth optimization to super-reflexive Banach spaces [3]. The δ -subharmonic where first given a systematic treatise in [14]. The class δ -plurisubharmonic functions were studied by Cegrell [4], and Kiselman [13], where the topology was defined by neighbourhood basis of the form $(U \cap \mathcal{PSH}) - (U \cap \mathcal{PSH})$, U a neighbourhood of the origin L^1_{loc}.

In [16], the authors gave a subset of *m*-subharmonic functions, the subclasses $\delta\mathcal{E}_m^0(\Omega)$ and $\delta(SH_m \cap L^\infty)(\Omega)$ of $\delta - SH_m(\Omega)$. In this paper we will give and study a subset of delta *m*-subharmonic functions, $\delta\mathcal{F}_p^m(\Omega)$.

For convenience we will denote the class of negative *m*-subharmonic functions on a domain Ω by $\mathcal{SH}_m^-(\Omega)$, and as in [9] we will denote the class of bounded *m*-subharmonic functions with boundary value zero and finite total *m*-Hessian mass by $\mathcal{E}_m^0(\Omega)$.

For the notation of the so called *energy class*

$\mathcal{F}_m(\Omega)$ on a hyperconvex domain Ω we refer to the paper [9]. As for now we remind the reader that the generalized complex m-Hessian operator is well defined in $\mathcal{F}_m(\Omega)$ and functions from $\mathcal{F}_m(\Omega)$ has finite total m-Hessian mass.

The paper is organized as follows. Beside the introduction the paper has two sections. In Section 2 we recall the definitions and results concerning to m-subharmonic functions which were introduced and investigated intensively in recent years by many authors, see [15],... We also recall the Cegrell classes of m-subharmonic functions $\mathcal{F}_m(\Omega)$ and $\mathcal{E}_m(\Omega)$ introduced and studied in [9] and the class $\mathcal{F}_\chi^m(\Omega)$ introduced and studied in [12]. Final, in Section 3 we construct a locally convex topology on the vector space $\delta\mathcal{F}_p^m, p \geq 0$.

2. Preliminairies

2.1. m-Hessians

Let Ω be a hyperconvex domain in $\mathbb{C}^n$. For a twice continuously differentiable real function $u \in C^2(\Omega)$, the secondorder differential at a fixed point $z_0 \in \Omega$

$$dd^c u = \frac{i}{2}\sum_{j,k}^{n} u_{j,\bar{k}}\, dz_j \wedge d\bar{z}_k$$

is a Hermitian quadratic form. After an appropriate unitary transformation of coordinates, it reduces to the diagonal form

$$dd^c u = \frac{i}{2}[\lambda_1 dz_1 \wedge d\bar{z}_1 + \cdots + \lambda_1 dz_n \wedge d\bar{z}_n],$$

where $\lambda_1(u), \ldots, \lambda_n(u)$ are the eigenvalues of the Hermitian matrix $(u_{j,\bar{k}})$, which are real, i.e.. $\lambda(u) = (\lambda_1(u), \ldots, \lambda_n(u)) \in \mathbb{R}^n$. It is easy to see that

$$(dd^c u)^m \wedge \beta^{n-m} = m!(n-m)!H_m(u)\beta^n, \forall 1 \leq m \leq n$$

holds in the sense of currents.

where $$H_m(u) = \sum_{1 \leq j_1 < \cdots < j_m \leq n} \lambda_{j_1} \ldots \lambda_{j_m}$$ is the Hessian of order m of the vector $\lambda(u) = (\lambda_1(u), \ldots, \lambda_n(u)) \in \mathbb{R}^n$. Thus, the operator $(dd^c u)^m \wedge \beta^{n-m}, \forall 1 \leq m \leq n$, for twice continuously differentiable functions is related to the Hessian of the vector $\lambda = (\lambda_1, \ldots, \lambda_n)$.

2.2. m-subharmonic functions

We recall the class of m-subharmonic functions introduced and investigated in [2] recently. For $1 \leq m \leq n$, we define

$$\hat{\Gamma}_m = \{\eta \in \mathbb{C}_{(1,1)} : \eta \wedge \beta^{n-1} \geq 0, \ldots, \eta^m \wedge \beta^{n-m} \geq 0\},$$

where $\mathbb{C}_{(1,1)}$ denotes the space of (1,1) -forms with constant coefficients.

Definition 2.1. Let u be a subharmonic function on an open subset $\Omega \subset \mathbb{C}^n$. Then u is said to be a m-subharmonic function on Ω if for every $\eta_1, \ldots, \eta_{m-1}$ in $\hat{\Gamma}_m$ the inequality

$$dd^c u \wedge \eta_1 \wedge \ldots \wedge \eta_{m-1} \wedge \beta^{n-m} \geq 0,$$

By $SH_m(\Omega)$ (resp. $SH_m^-(\Omega)$) we denote the cone of m-subharmonic functions (resp. negative m-subharmonic functions) on Ω. Before formulating basic properties of m-subharmonic, we recall the following (see [2]).

For $\lambda = (\lambda_1, \ldots, \lambda_n) \in \mathbb{R}^n$ and $1 \leq m \leq n$ define

$$S_m(\lambda) = \sum_{1 \leq j_1 < \cdots < j_m \leq n} \lambda_{j_1} \cdots \lambda_{j_m}.$$

Set

$$\Gamma_m = \{S_1 \geq 0\} \cap \{S_2 \geq 0\} \cap \cdots \cap \{S_m \geq 0\}.$$

By $\mathcal{H}$ we denote the vector space of complex hermitian $n \times n$ matrices over $\mathbb{C}$. For $A \in \mathcal{H}$ let $\lambda(A) = (\lambda_1, \ldots, \lambda_n) \in \mathbb{R}^n$ be the eigenvalues of A. Set

$$\tilde{S}_m(A) = S_m(\lambda(A)).$$

As in [11] we define

$$\tilde{\Gamma}_m = \{A \in \mathcal{H} : \lambda(A) \in \Gamma_m\} = \{\tilde{S}_1 \geq 0\} \cap \cdots \cap \{\tilde{S}_m \geq 0\}.$$

Example 2.2. Let $u(z_1,z_2,z_3,z_4)=5\|z_1\|^2+9\|z_2\|^2-\|z_3\|^2-\|z_4\|^2$. It follows from (01), we see that $u\in SH_2(\mathbb{C}^3)$. However, u is not a plurisubharmonic function in $\mathbb{C}^4$ because the restriction of u on the line $(0,0,z_3,0)\cup(0,0,0,z_4)$ is not subharmonic.

Now as in [9] and [10], we define the complex Hessian operator of locally bounded m-subharmonic functions as follows.

Definition 2.3. Assume that $u_1,\ldots,u_p\in SH_m(\Omega)\cap L^\infty_{loc}(\Omega)$. Then the complex Hessian operator $H_m(u_1,\ldots,u_p)$ is defined inductively by

$$dd^cu_p\wedge\cdots\wedge dd^cu_1\wedge\beta^{n-m}=dd^c(u_p dd^cu_{p-1}\wedge\cdots\wedge dd^cu_1\wedge\beta^{n-m}).$$

From the definition of m-subharmonic functions and using arguments as in the proof of Theorem 2.1 in [1] we note that $H_m(u_1,\ldots,u_p)$ is a closed positive current of bidegree $(n-m+p,n-m+p)$ and this operator in continuous under decreasing sequences of locally bounded m-subharmonic functions. Hence, for $p=m$, $dd^cu_1\wedge\cdots\wedge dd^cu_m\wedge\beta^{n-m}$ is a nonnegative Borel measure. In particular, when $u=u_1=\cdots=u_m\in SH_m(\Omega)\cap L^\infty_{loc}(\Omega)$ the Borel measure

$$H_m(u)=(dd^cu)^m\wedge\beta^{n-m},$$

is well defined and is called the complex Hessian of u.

2.3. The $\mathcal{E}_m^0(\Omega)$ and $\mathcal{F}_m(\Omega)$ classes

Next, we recall the classes $\mathcal{E}_m^0(\Omega)$ and $\mathcal{F}_m(\Omega)$ introduced and investigated in [9]. First we give the following.

Let Ω be a bounded domain in $\mathbb{C}^n$. Ω is said to be m-hyperconvex if there exists a continuous m-subharmonic function $u:\Omega\to\mathbb{R}^-$ such that $\Omega_c=\{u<c\}\Subset\Omega$ for every $c<0$. As above, every plurisubharmonic function is m-subharmonic with $m\geq 1$ then every hyperconvex domain in $\mathbb{C}^n$ is m-hyperconvex. Let $\Omega\subset\mathbb{C}^n$ be a m-hyperconvex domain. Set

$$\mathcal{E}_m^0=\mathcal{E}_m^0(\Omega)=\{u\in SH_m^-(\Omega)\cap L^\infty(\Omega):\lim_{z\to\partial\Omega}u(z)=0,\ \int_\Omega H_m(u)<\infty\},$$

$$\mathcal{F}_m=\mathcal{F}_m(\Omega)=\left\{u\in SH_m^-(\Omega):\exists\ \mathcal{E}_m^0\ni u_j\searrow u,\ \sup_j\int_\Omega H_m(u_j)<\infty\right\},$$

$$\mathcal{E}_m=\mathcal{E}_m(\Omega)=\left\{u\in SH_m^-(\Omega):\forall z_0\in\Omega,\exists\ \text{a neighborhood}\ \omega\ni z_0,\ \text{and}\ \mathcal{E}_m^0\ni u_j\searrow u\ \text{on}\ \omega,\ \text{and}\ \sup_j\int_\Omega H_m(u_j)<\infty\right\},$$

where $H_m(u)=(dd^cu)^m\wedge\beta^{n-m}$ denotes the Hessian measure of $u\in SH_m^-(\Omega)\cap L^\infty(\Omega)$. From Theorem 3.14 in [9] it follows that if $u\in\mathcal{E}_m(\Omega)$, the complex Hessian $H_m(u)=(dd^cu)^m\wedge\beta^{n-m}$ is well defined and is a Radon measure on Ω. On the other hand, by Remark 3.6 in [9] we may give the following description of the class $\mathcal{E}_m(\Omega)$:

$$\mathcal{E}_m=\mathcal{E}_m(\Omega)=\left\{u\in SH_m^-(\Omega):\forall\ U\Subset\Omega,\exists\ v\in\mathcal{F}_m(\Omega),\ v=u\ \text{on}\ U\right\}.$$

3. The topology and *m*-Hessian on $\delta\mathcal{E}_p^m$ class

3.1. Weighted energy classes of SH_m functions

The classes of SH_m functions were introduced and investigated by Hung in [12]

$$\mathcal{F}_\chi^m = \left\{ \varphi \in SH_m^-(\Omega) : \exists\ \mathcal{E}_0^m \ni \varphi_j \searrow \varphi, \sup_{j\geq 1} \int_\Omega \chi(\varphi_j) H_m \varphi_j)^n < +\infty \right\}.$$

When we take $\chi(t) = (-t)^p, t \in \mathbb{R}^+, p \geq 0,$ we have the following class of m-subhamornic functions

$$\mathcal{F}_p^m = \left\{ \varphi \in SH_m^-(\Omega) : \exists\ \mathcal{E}_0^m \ni \varphi_j \searrow \varphi, \sup_{j\geq 1} \int_\Omega (-\varphi_j)^p H_m(\varphi_j)^n < +\infty \right\}.$$

Now, by the definition of this class, we have the following result.

Theorem 3.1. Forall $p \geq 0,$ then class $\mathcal{F}_p^m$ has the following properties:

i) $\mathcal{F}_p^m$ is a convex cone set, i.e. if $\varphi, \psi \in \mathcal{F}_p^m$ and $a, b \geq 0$ then $a\varphi + b\psi \in \mathcal{F}_p^m$;

ii) $\mathcal{F}_p^m$ satisfies the max property, i.e. if $\varphi \in \mathcal{F}_p^m$ and $\psi \in SH_m^-(\Omega)$ then $\max(\varphi, \psi) \in \mathcal{F}_p^m$.

Proof. i) First, we prove that if $u \in \mathcal{F}_p^m(\Omega)$ then $\alpha u \in \mathcal{F}_p^m(\Omega)$. Indeed, let $\{u_j\} \subset \mathcal{E}_m^0(\Omega)$, $u_j \searrow u$ on Ω with

$$\sup_j \int_\Omega (-u_j)^p (dd^c u_j)^m \wedge \beta^{n-m} < \infty.$$

It is clear that $\{\alpha u_j\} \subset \mathcal{E}_m^0(\Omega)$, $\alpha u_j \searrow \alpha u$ on Ω. Moreover, since

$$\sup_j \int_\Omega (-\alpha u_j)^p (dd^c \alpha u_j)^m \wedge \beta^{n-m} = \alpha^{m+p} \sup_j \int_\Omega (-u_j)^p (dd^c u_j)^m \wedge \beta^{n-m} < \infty.$$

Hence $\alpha u \in \mathcal{F}_p^m(\Omega)$. By the above proof, we can assume that $\alpha + \gamma = 1$.

Let $\{u_j\}, \{v_j\} \subset \mathcal{E}_m^0(\Omega)$, $u_j \searrow u$ on Ω, $v_j \searrow u$ on Ω,

$$\sup_j \int_\Omega (-u_j)^p (dd^c u_j)^m \wedge \beta^{n-m} < \infty$$

and

$$\sup_j \int_\Omega (-v_j)^p (dd^c v_j)^m \wedge \beta^{n-m} < \infty.$$

By Lemma 2 in [12], we have

$$\sup_j \int_\Omega (-\alpha u_j - \gamma v_j)^p (dd^c(\alpha u_j + \gamma v_j))^m \wedge \beta^{n-m}$$
$$\leq 2^{m+1} \left(\sup_j \int_\Omega (-u_j)^p (dd^c u_j)^m \wedge \beta^{n-m} + \sup_j \int_\Omega (-v_j)^p (dd^c v_j)^m \wedge \beta^{n-m} \right) < \infty$$

It is clear that $\mathcal{F}_p^m$ satisfies the condition ii). Hence, the desired conclusion follows.

3.2. The $\delta\mathcal{E}_p^m$ class

Now we shall introduce the space $\delta\mathcal{F}_p^m$ and give some necessary elements that will be used to construct the topology on this space. We set

$$\delta\mathcal{F}_p^m = \mathcal{F}_p^m - \mathcal{F}_p^m = \{u \in \mathrm{L}^1_{\mathrm{loc}}(\Omega) : \exists v, w \in \mathcal{F}_p^m, u = v - w\}.$$

By Theorem 3.1, the class $\mathcal{F}_p^m$ is a convex cone and so $\delta\mathcal{F}_p^m$ is a vector space.

For $u \in \mathcal{F}_p^m$ we set

$$e_p(u) = \int_\Omega (-u)^p H_m(u).$$

For each $k \in \mathbb{N}$ we set

$$U_k = \{u = v - w : v, w \in \mathcal{F}_m^p, e_p(v) < \frac{1}{k}, e_p(w) < \frac{1}{k}\}.$$

In the next section, we are going to study some properties of the family of subsets $U_k, k \in \mathbb{N}$. Then we will construct the topology on the vector space $\delta\mathcal{F}_m^p$ from that family.

Now we equip once topology for the space $\delta\mathcal{F}_p^m$. Infact, we will prove that the vector space $\delta\mathcal{F}_p^m$ is a locally convex topological space, and moreover is Frechet space. First, we need some lemmas.

Lemma 3.2. The set U_k is a balanced subset in the vector space $\delta\mathcal{F}_p^m$, i.e. $\forall u \in U_k$, we have $au \in U_k, \forall |a| \le 1$.

Proof. Given $v \in \mathcal{F}_p^m$ and $0 \le a \le 1$. We have

$$e_p(av) = \int_\Omega (-av)^p H_m(av) = a^m \int_\Omega (-av)^m H_m(v)$$
$$\le \int_\Omega (-v)^p H_m(v) = e_\chi(v).$$

From this we infer that U_k is a balanced set.

Lemma 3.3. The set U_k is an absorbing subset in the vector space $\delta\mathcal{F}_p^m$, i.e. $\forall u \in \delta\mathcal{F}_p^m, \exists \epsilon > 0$ such that $au \in U_k, \forall |a| < \epsilon$.

Proof. First, given $v \in \mathcal{F}_p^m$ and $a \ge 1$ from the result above we have

$$e_p(av) = \int_\Omega (-av)^p H_m(av) = a^m \int_\Omega (-v)^p H_m(v)$$
$$\le c e_p(v) \cdot$$

From this result we imply that U_k is a absorbing set.

Theorem 3.4. In the class $\mathcal{F}_p^m$, we have the following estimates:

i) If $\varphi, \psi \in \mathcal{F}_p^m$ then

$$e_p(\varphi + \psi) \le 2^{2m} c^2 \left[e_p(\varphi) + e_p(\psi) \right].$$

ii) If $\varphi, \psi \in \mathcal{F}_p^m$ are such that $\varphi \ge \psi$ then $e_p(\varphi) \le 2^m c e_p(\psi)$.

Proof. By the definition of the class $\mathcal{F}_p^m$ it is enough to prove the proposition when $\varphi, \psi \in \mathcal{E}_0^m$

i) First, as in the proof of Proposition 3.4 in [8], we have

$$\int_{\{\varphi < -t\}} H_m(\varphi) \le t^m cap(\{\varphi < -t\}), \forall \varphi \in \mathcal{E}_0^m \tag{1}$$

$$t^m cap(\{\varphi < -2t\}) \le \int_{\{\varphi < -t\}} H_m(\varphi), \forall t > 1, \forall \varphi \in \mathcal{E}_0^m. \tag{2}$$

We have

$$e_p(\varphi) = \int_\Omega (-\varphi)^p H_m(\varphi) = \int_0^{+\infty} -p(-t)^{p-1} \int_{\{\varphi < -t\}} H_m(\varphi) dt.$$

We set $e_0(\varphi) = \int_\Omega H_m(\varphi)$. By the Lemma 2.5 in [8], we have

$$e_0(\varphi + \psi)^{\frac{1}{m}} \le e_0(\varphi)^{\frac{1}{m}} + e_0(\psi)^{\frac{1}{m}}.$$

We set

$$\tilde{e}_p(\varphi) = \int_0^{+\infty} -p(-t)^{p-1} t^m cap(\{\varphi < -t\}) dt.$$

Applying the formula (1) we infer

$$e_p(\varphi) \le \tilde{e}_p(\varphi), \forall \varphi \in \mathcal{E}_0^m. \tag{3}$$

Applying the formula (2), we have

$$\tilde{e}_p(\varphi) \le \int_0^{+\infty} -p(-t)^{p-1} 2^m \int_{\{\varphi < -\frac{t}{2}\}} H_m(\varphi) dt$$
$$= 2^m \int_0^{+\infty} -p(-t)^{p-1} \int_{\{\varphi < -\frac{t}{2}\}} H_m(\varphi) dt$$
$$= 2^{m+p-1} \int_0^{+\infty} -p(-t)^{p-1} \int_{\{\varphi < -t\}} H_m(\varphi) dt.$$

So we imply
$$\begin{aligned}\tilde{e}_p(\varphi) &\le 2^{m+p-1}\int_0^{+\infty} -p(-t)^{p-1}\int_{\{\varphi<-t\}} H_m(\varphi)dt\\ &\le 2^m e_{2p}(\varphi)\\ &= 2^m\int_\Omega (-2\varphi)^p H_m(\varphi)\\ &\le 2^m c\int_\Omega (-\varphi)^p H_m(\varphi)\\ &= 2^m ce_p(\varphi).\end{aligned}$$

Therefore we have

$$\tilde{e}_p(\varphi) \le 2^m ce_p(\varphi). \tag{4}$$

For every $a \in [0,1]$ we have

$$\begin{aligned}\tilde{e}_p((1-a)\varphi + a\psi) &= \int_0^{+\infty} -p(-t)^{p-1} t^m cap(\{(1-a)\varphi + a\psi < -t\})dt\\ &\le \int_0^{+\infty} -p(-t)^{p-1} t^m cap(\{\varphi < -t\}\cup\{\psi < -t\})dt\\ &\le \int_0^{+\infty} -p(-t)^{p-1} t^m [cap(\{\varphi < -t\}) + cap(\{\psi < -t\})]dt\\ &= \tilde{e}_p(\varphi) + \tilde{e}_p(\psi).\end{aligned}$$

The following inequalities are straightforward

$$\begin{aligned}e_0(\varphi+\psi) &\le (e_0(\varphi)^{\frac{1}{m}} + e_0(\psi)^{\frac{1}{m}})^m\\ &\le 2^{m-1}(e_0(\varphi)+e_0(\psi)),\\ e_0(\frac{\varphi+\psi}{2}) &\le \frac{1}{2}[e_0(\varphi)+e_0(\psi)].\end{aligned}$$

Using the results above we obtain the following estimations

$$\begin{aligned}e_p(\varphi+\psi) = e_p(2\cdot\frac{\varphi+\psi}{2}) &\le 2^m ce_p(\frac{\varphi+\psi}{2})\\ &\le 2^m c\tilde{e}_p(\frac{\varphi+\psi}{2})\\ &\le 2^m c\left[\tilde{e}_p(\varphi)+\tilde{e}_p(\psi)\right]\\ &\le 2^m c\left[2^m ce_p(\varphi)+2^m ce_p(\psi)\right]\\ &= 2^{2m}c^2[e_p(\varphi)+e_p(\psi)].\end{aligned}$$

So i) was proved.

ii) It is a consequence of (3) and (4).

Final, we shall prove $\delta\mathcal{F}_p^m$ is a locally convex topology space.

Theorem 3.5. The vector space $\delta\mathcal{F}_p^m$ is a locally convex topology.

Proof. It follows from the Theorem 3.4 that for every U_k $(k \ge 1)$ we can find q, for example, we can choose $q = k([2^{2m}c^2]+1$ such that the convex hull of the set U_q is contained in U_k.

On other hand, combined with Lemma 3.2 and Lemma 3.3, that the family $\mathcal{A}$ of convex hull of sets $U_k, k \ge 1$ is a family of absorbing, balanced, convex sets in the vector space

$\delta\mathcal{F}_p^m$. So there is a locally convex topology on this space such that the family $\mathcal{A}$ becomes a neighbourhood basis of origin.

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MỘT SỐ TÍNH CHẤT TÔPÔ CỦA LỚP $\delta\mathcal{F}_p^m$

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Trường Đại học Tây Bắc

Tóm tắt: *Trong bài báo này, chúng tôi sẽ đưa ra và nghiên cứu một lớp con của lớp delta m-điều hòa dưới,* $\delta\mathcal{F}_p^m(\Omega), p \geq 0$*. Sau đó chúng tôi sẽ trang bị một topo cho không gian này và chứng minh rằng không gian vector* $\delta\mathcal{F}_p^m$ *là lồi địa phương, hơn nữa nó còn là một không gian Frechet.*

Từ khóa: *Hàm* δ *-đa điều hòa dưới, hàm m-điều hòa dưới, toán tử m-Hessian, toán tử Monge-Ampère, hàm trọng, không gian vector tôpô lồi địa phương.*

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