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# THERMODYNAMIC PROPERTIES OF REISSNER-NORDSTRÖM BLACK HOLE

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***Abstract:*** *The thermodynamic properties of matter outside of the 4-dimensional Reissner-Nordström (RN) charged black hole have been investigated. Has shown that matter have similar properties to Van der Walls fluid and with temperatures $T$ less than the critical temperature $T_c$ there exists a gas-liquid phase transition.*

***Keywords***: *thermodynamic properties, black hole, gas-liquid, phase transition.*

## 1. Introduction

Quantum mechanically, black holes have thermodynamic properties like ordinary statistical systems [1]. According to thermodynamics, one does not have to specify the position and the momentum of each molecule to characterize a thermodynamic system. The system can be characterized only by a few macroscopic variables such as temperature, entropy and pressure...

Thermodynamic properties of black holes have been studied for many years [2], [3], [4]... It has been shown that black hole spacetime is not only considered to be a thermodynamic standard variable like temperature and entropy, but also shows that it leads to abundant phase structures and many critical phenomena as the same as other known non- gravity thermodynamic systems in nature. In this paper we explore thermodynamic properties of the RN charged black hole.

The paper is organized as follows. Section II is the content of the article, in which presents the calculus steps to obtain the thermodynamic quantities as pressure, temperature and the results of numerical computation. The conclusion and outlook are presented in section III.

## 2. Content

### *2.1. Equations of state*

We start from the Lagrangian £ given by

$$16\pi G_N £ = R - \frac{12}{L^2} - \frac{1}{4}F_{\mu\nu}^2 - |\partial_\mu\psi - iQA_\mu\psi|^2 - m^2|\psi|^2, \qquad (1)$$

where $G_N$ being the Newton's universal gravitational constant; $R$ is Ricci scalar; $A_\mu$ and $\psi$ represent the gauge field and scalar field, respectively; $m$ is the mass of field $\psi$; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor; $L$ is the $AdS_4$ radius (related to the cosmological constant $\Lambda : \Lambda = -3/L^2$).

The corresponding action reads

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left(R - \frac{12}{L^2} - \frac{1}{4}F_{\mu\nu}^2 - |\partial_\mu\psi - iQA_\mu\psi|^2 - m^2|\psi|^2\right), \quad (2)$$

when $\psi = 0$ which provides the Reissner-Nordström (RN) charged black hole in four-dimensional anti de Sitter spacetime ($AdS_4$) with the metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2, \quad (3)$$

in which

$$f(r) = k - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{L^2}, \quad (4)$$

outside of the black hole.

Here $M$ and $Q$ are the mass and charge of black hole; $k$ stands for the spatial curvature. In (1.2) $d\Omega_2^2$ is the metric of the associated 2-dimensional manifold with constant curvature 2k. If k = 0 then $d\Omega_2^2$ is the line element of a plane. If k > 0, then $d\Omega_2^2$ is the metric of a two-sphere $S^2$ of radius $1/\sqrt{k}$. If k < 0, then $d\Omega_2^2$ is the metric of the hyperboloid with radius of curvature $1/\sqrt{-k}$, we will not consider this case .

To write entirely the metric tensor of (1.2) let begin with the metric tensor of $d\Omega_2^2$ as follows. The two-sphere $S^2$ is described by the equation

$$x_1^2 + x_2^2 + x_3^2 = a^2, a = 1/\sqrt{k}. \quad (5)$$

Imposing

$$x_1 = a\cos\alpha\cos\beta;\ x_2 = a\sin\alpha\cos\beta;\ x_3 = a\sin\beta, \quad (6)$$

it follow that

$$d\Omega_2^2 = \sum_{i=1}^{2} dx_i^2 = \frac{\cos^2\beta d\alpha^2 + \sin^2\beta d\beta^2}{k}. \quad (7)$$

Inserting (7) into (3) we arrive

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2\cos^2\beta}{k}d\alpha^2 + \frac{r^2\sin^2\beta}{k}d\beta^2 \quad (8)$$

which is of the form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu;\ \mu,\nu = 0,1,2,3, \quad (9)$$

in which

$$dx^0 = dt,\ dx^1 = \alpha,\ dx^2 = \beta,\ dx^3 = dr, \quad (10)$$

Indemtify terms of (8) with corresponding terms of (1.8) we deduce the expressions of metric tensor $g_{\mu\nu}$

$$g_{tt} = -f(r),\ g_{\alpha\alpha} = \frac{r^2\cos^2\beta}{k},\quad g_{\beta\beta} = \frac{r^2\sin^2\beta}{k},\ g_{rr} = 1/f(r),\ g^{tt} = -1/f(r),$$

$$g^{\alpha\alpha} = \frac{k}{r^2\cos^2\beta},\quad g^{\beta\beta} = \frac{k}{r^2\sin^2\beta},\ g^{rr} = f(r),\ g_{\mu\nu} = g^{\mu\nu} = 0\ \ if\ \ \mu \neq \nu. \quad (11)$$

The determinant of the metric tensor is defined as

$$g = \det|g_{\mu\nu}| = -\frac{r^4\cos^2\beta\sin^2\beta}{k^2} \to \sqrt{-g} = \frac{r^2\cos\beta\sin\beta}{k}. \quad (12)$$

Next the radius horizon $r_+$ is defined as the larger root of $f(r_+) = 0$. So

$$f(r_+) = k - \frac{2M}{r_+} + \frac{Q^2}{r_+^2} + \frac{r_+^2}{L^2} = 0, \quad (13)$$

from which we derive

$$M = \frac{r_+}{2}\left[k + \frac{Q^2}{r_+^2} + \frac{r_+^2}{L^2}\right]. \quad (14)$$

Inserting (14) into (4) we obtain

$$f(r)=k\left(1-\frac{r_+}{r}\right)+\frac{Q^2}{r^2}\left(1-\frac{r_+}{r}\right)+\frac{r^2}{L^2}\left(1-\frac{r_+^3}{r^3}\right). \quad (15)$$

The Hawking temperature reads

$$T=\frac{f'(r_+)}{4\pi} \quad (16)$$

Combining (4), (14) and (16) we obtain

$$T=\frac{1}{2\pi}\left(-\frac{Q^2}{r_+^3}+\frac{M}{r_+^2}+\frac{r_+}{L^2}\right)=\frac{1}{4\pi r_+}\left(k-\frac{Q^2}{r_+^2}+\frac{3r_+^2}{L^2}\right). \quad (17)$$

The pressure of black hole is defined as [5]:

$$P=-\frac{\Lambda}{8\pi}=\frac{3}{8\pi}\frac{1}{L^2}. \quad (18)$$

In the case of a RN black hole the volume is given by

$$V=\frac{4}{3}\pi r_+^3, \quad (19)$$

Eqs. (17), (18) and (19) constitute the equations of state governing all thermodynamical processes.

**2.2. Thermodynamic properties**

In order to get insight into the thermodynamic properties of RN black hole one has to carry out a numerical study. In the figures below, dimensionless quantities are used.

First of all, let us study the state equation $P(V,T)$. Combining (17), (18) and (19) we arrive

$$P/P_c=\frac{1-6(V/V_c)^{2/3}+8(T/T_c)(V/V_c)}{3(V/V_c)^{4/3}}, \quad (20)$$

where

$$P_c=\frac{k^2}{96\pi Q^2};\quad V_c=\frac{8\sqrt{6}\pi Q^3}{k^{3/2}};\ \text{and}\quad T_c=\frac{k^{3/2}}{3\sqrt{6}\pi Q}. \quad (21)$$

Now we draw the volume dependence of the pressure at several values of temperature. Figure.1 represents the behaviour of isotherms in the $P-V$ diagram. As is seen from this figure they have a similar pattern to isotherms of the Van der Waals system. Moreover, for temperatures $T<T_c$ there exits a minimum of pressure. It shows that there is a liquid–gas phase transition outside black hole. In contrast, with $T>T_c$ there will be no phase transition-the system is always gaseous. At $T=T_c$ isotherms have only inflection points, so $T=T_c$ is critical temperature.

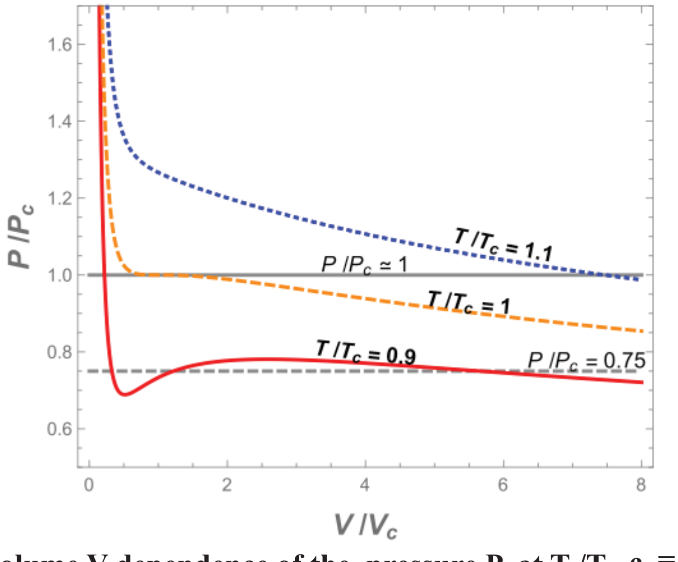


**Figure 1: The volume V dependence of the pressure P at T /T c = 0, 9; 1, 0; 1, 1.**

Next the radius horizon dependence of the Hawking temperature is concerned. Basing on the (17) and (18) we are able to write

$$T/T_c = \frac{-1+6(r_+/r_{c+})^2+3(P/P_c)(r_+/r_{c+})^4}{8(r_+/r_{c+})^3}, \qquad (22)$$

where

$$r_{c+} = \frac{Q\sqrt{6}}{k^{1/2}}. \qquad (23)$$

Then we draw the radius horizon $r_+$ dependence of the temperature $T$ at several values of the pressure, which given in Figure.2.

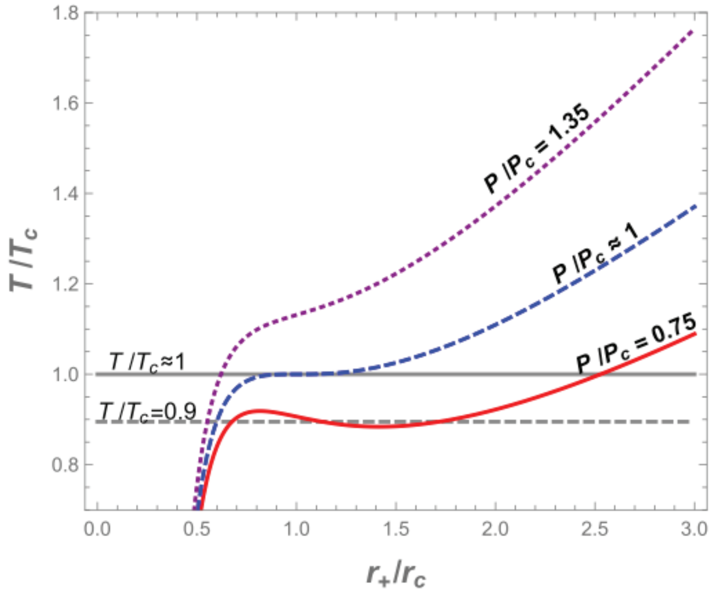


**Figure 2: The radius horizon r+ dependence of the temperature T at P /Pc = 0, 75; 1, 00; 1, 35.**

Using the expression of entropy $\zeta = \pi r_+^2$ we are able to rewrite (22) as

$$T_c = \frac{-1+6(\zeta/\zeta_c)+3(P/P_c)(\zeta/\zeta_c)^2}{8(\zeta/\zeta_c)^{3/2}}, \qquad (24)$$

where

$$\zeta_c = \frac{6\pi Q^2}{k}. \qquad (25)$$

Basing on (24) we draw the entropy dependence of the temperature. Figure.3 represents the curves of $T$ vs $\zeta$ at several values of the pressure.

From Figs.2 and 3 it is clear that there exists a gas-liquid phase transition when $T < T_c$ and $T_c$ is critical temperature corresponding to above mentions.

**3. Conclusion and Outlook**

Let us now summarize the main results presented in the previous sections. From the metric of the RN charged black hole we have found expressions for Hawking temperature and pressure outside of the black hole. Based on these expressions, we have calculated numerically to examine thermodynamic properties and obtained the following result:

* At temperatures below the critical temperature $T_c$, matter outside the black hole can be gaseous or liquid. In contrast, with temperatures greater than $T_c$, matter is always gaseous. Thus, what kind of this matter here is a question for further studies.

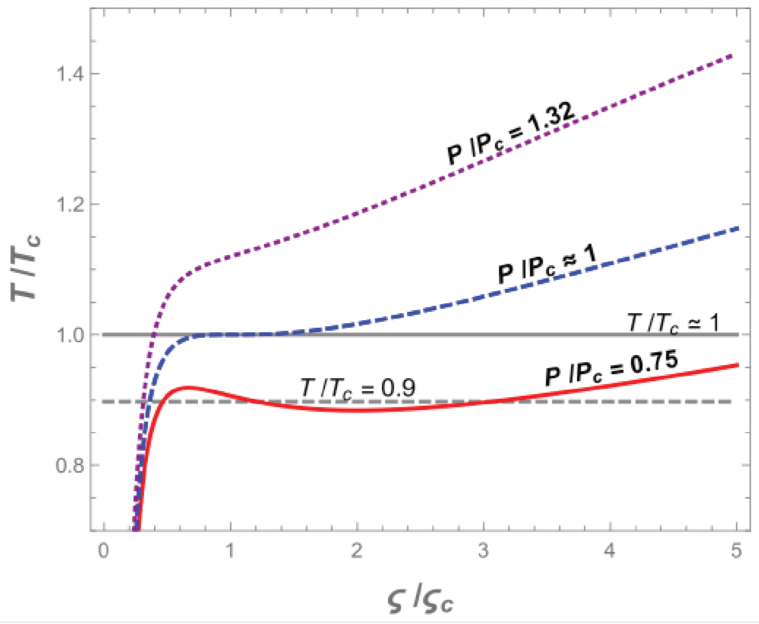


**Figure 3: The entropy ζ dependence of the temperature T at P /Pc = 0, 75; 1, 00; 1, 32.**

* With temperatures less than the critical temperature $T_c$ there exists a gas-liquid phase transition of matter. That is consistent with the results already obtained in [5].

To conclude, we would like to emphasize that the above results are obtained only with $k > 0$. For comprehensive conclusions, it is necessary to consider with $k < 0$ or $k = 0$. This is the our research next.

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# THERMODYNAMIC PROPERTIES OF REISSNER-NORDSTRÖM BLACK HOLE

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***Tóm tắt:*** *Các tính chất nhiệt động lực học của vật chất bên ngoài hố đen tích điện Reissner-Nordström (RN) 4 chiều đã được nghiên cứu. Nghiên cứu đã chỉ ra rằng vật chất ở bên ngoài hố đen tích điện RN 4 chiều có các tính chất tương tự như chất lỏng van der Waals và ở nhiệt độ T thấp hơn nhiệt độ tới hạn Tc tồn tại một chuyển pha khí lỏng.*

***Từ khóa:*** *Các tính chất nhiệt động, hố đen, khí-lỏng, chuyển pha.*

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