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CASIMIR-TYPE FORCE OF AN IDEAL BOSE-EINSTEIN CONDENSATE GAS IN BROKEN SYMMETRY PHASE

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Abstracts: *We consider the Casimir-type effect of an ideal Bose-Einstein condensate (BEC) gas, which is confined by two parallel plates in the (x, y)-plane and separated by distance ℓ along z-direction for any boundary conditions (BCs). In which the Casimir-type energy is proportional to ℓ^{-2} and the resulting in Casimir-type force decay as ℓ^{-3}.*

Keywords: *Bose gas; Casimir force; Finite-size effect.*

I. INTRODUCTION

Beside studying of the attractive Casimir force [1,2], researching the repulsive Casimir force is an interesting subject. They were considered in many systems, such as electromagnetic field [3], massless scalar field $[4-9]$ and BEC(s) gas $[10-25]$. In the ideal BEC(s) area, the Casimir-type effect was investigated in both grand canonical ensemble (GCE) $[10, 13-15]$ and canonical ensemble (CE) [11,12]. For imperfect BEC(s), the effect was first mentioned in [16] for Dirichlet BC, after that the forces corresponding periodic BC [17, 18], Robin BC [22], Zaremba BC, and anti periodic BC [25] have been discovered, respectively. Although the Casimir-type effect in an ideal BEC confined between two parallel plates has been investigated by many authors $[10-15]$. But this paper aims to show a simpler and more explicit way to find out the Casimir-type energy as well as Casimir-type force, to our acknowledgment. Our paper is organized as follows: In Sec.II, we introduce the thermodynamical grand canonical energy of a weakly interacting Bose gas. The Casimir effect of a perfect BEC gas is substantiated in Sec.III. Discussions and Conclusions are given in Sec.IV and Sec.V, respectively, to close the paper.

II. THE THERMODYNAMICAL GRAND CANONICAL ENERGY IN 1-LOOP APPROXIMATION

The Casimir-type effect in BEC arising from finite-size effect of grand canonical potential [11]. In this section, the thermodynamical grand canonical energy of a weakly interacting Bose gas will be established. Let us begin with the Lagrangian density of a weakly interacting Bose gas [26]

$$\mathcal{L} = -\phi^* \left(i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \right) \phi(\vec{r}, t) + V, \quad (1)$$

where

$$V = -\mu |\phi(\vec{r}, t)|^2 + \frac{\lambda}{2} |\phi(\vec{r}, t)|^4 \quad (2)$$

is density of interacting potential, $\hbar$ and μ being the Planck constant and chemical potential. The coupling constant λ characterizes repulsive pair interaction strength between identical atoms, which is dependent on the s-wave scattering length a_s, and atomic mass m within formula $\lambda = 4\pi h^2 a_s/m > 0$ [28]. The order parameter is determined by the expectation value of the field operator $\phi(\vec{r}, t)$.

Firstly, one considers the tree approximation, in which the quantum fluctuation is neglected. Let ϕ_0 be the expectation value of the field operator, minimizing the potential density (2) with respect to the field operator leads to the gap equation

$$\phi_0 (-\mu + \lambda \phi_0^2) = 0 \quad (3)$$

In the broken symmetry phase, above equation gives

$$\phi_0^2 = \mu/\lambda \quad (4)$$

Our system is considered in connection to a bulk reservoir of condensate, so that the chemical potential can be read as [27]

$$\mu = \lambda \rho_c \left[1 + \mathcal{O}\left(\sqrt{\rho_c a_s^3} \right) \right] \quad (5)$$

where ρ_c is bulk density of the condensate. The quantity $\rho_c a_s^3$ is called gas parameter that satisfies the diluteness condition $\rho_c a_s^3 \ll 1$ [27], so higher level terms of the gas parameter, and quantum fluctuation is ignored. Combining Eqs. (4) and (5) one finds the condensate density in the lowest level approximation is

$$\rho_c = \phi_0^2 \quad (6)$$

Next, the problem is considered in one loop approximation, this means that the quantum fluctuation is taken into account. Denote two real fields ϕ_1 and ϕ_2 corresponding to the quantum fluctuations, the field operator can be expanded in term of the order parameter ϕ_0 and the fluctuation fields as [27]

$$\phi \to \phi_0 + \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \quad (7)$$

Plugging (7) into (1), the interaction Lagrangian density in the one-loop approximation is found out

$$\mathcal{L}_{int} = \frac{\lambda}{2}\phi_0\phi_1(\phi_1^2 + \phi_2^2) \quad (8)$$

From(8) and (3) one has the inversion propagator in momentum space

$$\mathcal{D}^{-1}(k) = \begin{pmatrix} \frac{\hbar^2k^2}{2m} + 2\lambda\phi_0^2 & \omega_n \\ -\omega_n & \frac{\hbar^2k^2}{2m} \end{pmatrix} \quad (9)$$

in which $\vec{k}$ being the wave vector. The Matsubara frequency for boson is defined as $\omega_n = 2\pi n\beta^{-1}$, $\beta = k_B^{-1}T^{-1}$, $(n = 0, \ \pm 1, \pm 2 \dots)$, k_B being Boltzmann constant. Recast that the Bogoliubov dispersion relation can be obtained by vanishing of the determinant of the inversion propagator [29]

$$\det \mathcal{D}^{-1}(k) = 0 \quad (10)$$

Dispersion relation produced from Eq.(10) is

$$\mathcal{E}(k) = \sqrt{\frac{\hbar^2k^2}{2m}\left(\frac{\hbar^2k^2}{2m} + 2\lambda\phi_0^2\right)} \quad (11)$$

Eq. (11) shows that there is a Goldstone boson associating with $U(1)$ breaking. In long wavelength limit, the dispersion relation (11) becomes $\mathcal{E}(k) \approx \hbar v_s k$, in which $v_s = \sqrt{\lambda/m}\phi_0$ is the sound speed.

Using the interaction Lagrangian density (8), the density of thermodynamical potential has the form [21, 27]

$$\Omega = -\mu\phi_0^2 + \frac{\lambda}{2}\phi_0^4 + \frac{1}{2}\int_\beta \mathrm{Tr}\ \ln \mathcal{D}^{-1}(k) \ , \quad (12)$$

the notation $\int_\beta f(k) = \frac{1}{\beta}\sum_{n=-\infty}^{+\infty}\int \frac{d^3\vec{k}}{(2\pi)^3} f(\omega_n, \vec{k})$ is employed. In order to perform summation over the Matsubara frequency, we use the rule [30]

$$\frac{1}{\beta}\sum_{n=-\infty}^{+\infty} \ln\left[\omega_n^2 + E^2(k)\right] = E(k) + \frac{2}{\beta}\ln\left[1 - e^{-\beta E(k)}\right].$$

Thus the third term in right hand side of Eq. (12) reduces to

$$\frac{1}{2}\int_\beta \mathrm{Tr}\ \ln D^{-1}(k) = \frac{1}{2}\int \frac{d^3\vec{k}}{(2\pi)^3}\left\{\mathcal{E}(\vec{k}) + \frac{2}{\beta}\ln\left[1 - e^{-\beta\mathcal{E}(\vec{k})}\right]\right\}. \quad (13)$$

Substituting (13) into (12) one arrives the relation of the thermodynamical grand canonical energy density

$$\Omega = -\mu\phi_0^2 + \frac{\lambda}{2}\phi_0^4 + \frac{1}{2}\int \frac{d^3\vec{k}}{(2\pi)^3}\mathcal{E}(\vec{k}) + \frac{1}{\beta}\int \frac{d^3\vec{k}}{(2\pi)^3}\ln\left[1 - e^{-\beta\mathcal{E}(\vec{k})}\right]. \quad (14)$$

The physical meaning of the right hand side of Eq. (14) is easy to recognized as following.

The two first terms

$$\Omega_{\mathrm{tree}} = -\mu\phi_0^2 + \frac{\lambda}{2}\phi_0^4 \quad (15)$$

characteristics the density of ground state energy in the tree approximation.

The two last terms due to the contribution of the fluctuations. In more detail, the third term

$$\Omega_0 = \frac{1}{2}\int \frac{d^3\vec{k}}{(2\pi)^3}\mathcal{E}(\vec{k}) \quad (16)$$

is the grand canonical energy density at zero temperature, which is produced from depletion of condensate, and the last term

$$\Omega_T = \frac{1}{\beta}\int \frac{d^3\vec{k}}{(2\pi)^3}\ln\left[1 - e^{-\beta\mathcal{E}(\vec{k})}\right] \quad (17)$$

is the thermal grand canonical energy density, which corresponds to the thermal fluctuations.

THE CASIMIR-TYPE EFFECT IN AN IDEAL BEC GAS

The broken symmetry phase of an ideal Bose gas occurs when temperature of the system below critical temperature $T_c^{(0)} = 3.31\hbar^2 n_0^{2/3}/mk_B$[28], where n_0 being the atoms density. In this section, the influence of the space compactification on the grand canonical energy of an ideal BEC gas is considered in $T < T_c^{(0)}$ regime. This implies that the system in phase with broken symmetry, and the chemical potential is annihilated. Thus the grand canonical ensemble defined by (17), with $\mathcal{E}(k) = h^2k^2/2m$. To deal with the Casimir effect, we assume that the system is confined betweem two parallel plates with the size $L_x \times L_y$ in the *(x,y)*-plane and separating at distance ℓ along $Oz-$ direction. In the "bulk" limit, $L_x \to \infty, L_y \to \infty, \ell \to \infty$, grand canonical energy of the system (17) has the form

$$\Omega_T = \frac{1}{2\pi^2\beta}\int_0^\infty dk\ k^2\ \ln\left[1-e^{-\frac{\lambda_B^2k^2}{4\pi}}\right] \quad (18)$$

here $\lambda_B = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}$ is the de Broglie wavelength.

Using Taylor series

$$\ln\left[1-e^q\right] = -\sum_{j=1}^\infty \frac{e^{-jq}}{j}, \quad (19)$$

(18) becomes

$$\Omega_T = -\frac{1}{2\pi^2\beta}\sum_{j=1}^\infty \int_0^\infty d\,kk^2\frac{e^{-j\frac{\lambda_B^2k^2}{4\pi}}}{j} \quad (20)$$

Perform integration over k, and then take summation over j one obtains the density of grand canonical energy

$$\Omega_{\text{bulk}}(T) = -\frac{\zeta[5/2]}{2\sqrt{2}\pi^{3/2}}\frac{k_B^{5/2}T^{5/2}m^{3/2}}{\hbar^3} \quad (21)$$

In the slab limit, this means that $L_x \to \infty$, $L_y \to \infty$, while ℓ is finite, which leads to k_z wave vector component is quantized as following

$$\int\frac{d^3\vec{k}}{(2\pi)^3} \to \frac{1}{\ell}\sum_i\int_0^\infty \frac{dk_\perp k_\perp}{2\pi}, k^2 \to k_\perp^2 + k_i^2, k_i = (i+1/2)/L, \quad (22)$$

in which [14]

$$\begin{cases} L = \ell/2\pi, i = 0, \pm1, \pm2 \ldots \text{for anti-periodic BC,} \\ L = \ell/\pi, i = 0,1,2 \ldots \text{ for Zaremba BC.} \end{cases}$$

Using Eq.(17), quantization condition (22), and the rule (19) we arrive at the grand canonical energy per an unit area of plate

$$\Omega^{(\text{A,id})}[T,\ell] = -\frac{1}{\pi\beta}\sum_{i=0}^\infty\sum_{j=1}^\infty\int_0^\infty dk_\perp k_\perp\frac{e^{-\frac{j}{4\pi}\left(\lambda_B^2k_\perp^2+\frac{\lambda_B^2(i+1/2)^2}{L^2}\right)}}{j}, \quad (23)$$

$$\Omega^{(\text{Z,id})}[T,\ell]) = -\frac{1}{2\pi\beta}\sum_{i=0}^\infty\sum_{j=1}^\infty\int_0^\infty dk_\perp\, k_\perp\frac{e^{-\frac{j}{4\pi}\left(\lambda_B^2k_\perp^2+\frac{\lambda_B^2(i+1/2)^2}{L^2}\right)}}{j}, \quad (24)$$

for anti-periodic BC and Zaremba BC, respectively. One can sees that (23) and (24) can be rewritten in the forms

$$\Omega^{(\text{A,id})}[T,\ell] = -\frac{1}{\pi\beta}\sum_{i=0}^\infty\sum_{j=1}^\infty\frac{1}{j}\int_0^\infty dk_\perp\, k_\perp\left(e^{-\frac{j}{4\pi}\left(\lambda_B^2k_\perp^2+\frac{\lambda_B^2i^2}{(2L)^2}\right)} - e^{-\frac{j}{4\pi}\left(\lambda_B^2k_\perp^2+\frac{\lambda_B^2i^2}{L^2}\right)}\right) \quad (25)$$

$$\Omega^{(\text{Z,id})}[T,\ell] = -\frac{1}{2\pi\beta}\sum_{i=0}^\infty\sum_{j=1}^\infty\frac{1}{j}\int_0^\infty dk_\perp\, k_\perp\left(e^{-\frac{j}{4\pi}\left(\lambda_B^2k_\perp^2+\frac{\lambda_B^2i^2}{(2L)^2}\right)} - e^{-\frac{j}{4\pi}\left(\lambda_B^2k_\perp^2+\frac{\lambda_B^2i^2}{L^2}\right)}\right). \quad (26)$$

To evaluate Casimir energy, we now consider the quantity

$$I_i^{(\text{id})}[T,\ell] = \sum_{i=0}^\infty\sum_{j=1}^\infty\frac{1}{j}\int_0^\infty dk_\perp\, k_\perp e^{-\frac{j}{4\pi}\left(\lambda_B^2k_\perp^2+\frac{\lambda_B^2i^2}{L^2}\right)}. \quad (27)$$

Perform integral over $k_\perp$, Eq.(27) becomes

$$I_i^{(\text{id})}[T,\ell] = \frac{2\pi}{\lambda_B^2}\sum_{i=0}^\infty\sum_{j=1}^\infty\frac{1}{j^2}e^{-j\frac{i^2\alpha^2}{4\pi}} = \frac{2\pi}{\lambda_B^2}\left(\sum_{i=1}^\infty\sum_{j=1}^\infty\frac{1}{j^2}e^{-j\frac{i^2\alpha^2}{4\pi}} + \sum_{j=1}^\infty\frac{1}{j^2}\right). \quad (28)$$

By employing Euler-Maclaurin formula in the from [31]

$$\sum_{n=a}^b f[n] = \int_a^b f[x]dx + \frac{1}{2}(f[a]+f[b])$$

$$+\frac{1}{12}(f^{(1)}[b]-f^{(1)}[a|) - \frac{1}{720}(f^{(3)}[b] - f^{(3)}[a]) + \frac{1}{30240}(f^{(5)}[b]-f^{(5)}[a]+\ldots \quad (29)$$

one finds

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{1}{j^2}e^{-j\frac{i^2\alpha^2}{4\pi}} = \frac{\pi\zeta[5/21}{\alpha} - \frac{49}{60} + \frac{23\,\alpha^2}{1512\pi} + \mathcal{O}[\alpha^3] \quad (30)$$

In which $\alpha = \lambda_B/L$, in thermal equilibrium limit $\alpha \ll 1$ [11)]. The finite part of second term in the bracket of (28) is dropped out as following. With the help of gamma function definition, it is easy to find that

$$\sum_{j=1}^{\infty}\frac{1}{j^2} = \frac{1}{\Gamma[2]}\sum_{j=1}^{\infty}\int_0^{\infty} dt\, te^{-jt} \quad (31)$$

Using the rule (29) to evaluate the summation over j , and then perform integration over t one gets

$$\sum_{j=1}^{\infty}\frac{1}{j^2} = \frac{49}{30}. \quad (32)$$

Plugging (30) and (32) into (28) we arrive

$$I_i^{(\mathrm{id})}[T,\ell] = \frac{2\pi}{\lambda_B^2}\left(\frac{\pi\zeta\left[\frac{5}{2}\right]}{\alpha} + \frac{49}{60} + \frac{23\,\alpha^2}{1512\pi}\right). \quad (33)$$

By the same way above one find

$$I_{i/2}^{(\mathrm{id})}[T,\ell] = \sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\frac{1}{j}\int_0^{\infty} d\,k_\perp k_\perp e^{-\frac{j}{4\pi}\left(\lambda_B^2 k_\perp^2 + \frac{\lambda_B^2 i^2}{(2L)^2}\right)}$$

$$= \frac{2\pi}{\lambda_B^2}\left(\frac{\pi\zeta[5/2]}{(\alpha/2)} + \frac{49}{60} + \frac{23\,(\alpha/2)^2}{1512\pi}\right). \quad (34)$$

Subtracting (33) from (34) one gets

$$I_{i/2}^{(\mathrm{id})}[T,\ell] - I_i^{(\mathrm{id})}[T,\ell] = \frac{2\pi}{\lambda_B^2}\left(\frac{\pi\zeta[5/2]}{\alpha} - \frac{23\,\alpha^2}{2016\pi}\right). \quad (35)$$

Substituting (35) into (25), (26) we obtain

$$\Omega^{(A,id)}[T,\ell] = -\frac{1}{\pi\beta}\frac{2\pi}{\lambda_B^2}\left(\frac{\pi\zeta[5/2]}{\alpha} - \frac{23\alpha^2}{2016\pi}\right) = -\frac{\zeta[5/2]}{2\sqrt{2}\pi^{3/2}}\frac{k_B^{5/2}\tau^{5/2}m^{3/2}\ell}{\hbar^3} + \frac{23\pi k_B T}{252\,\ell^2}, \quad (36)$$

$$\Omega^{(Z,id)}[T,\ell] = -\frac{1}{2\pi\beta}\frac{2\pi}{\lambda_B^2}\left(\frac{\pi\zeta[5/2]}{\alpha} - \frac{23\,\alpha^2}{2016\pi}\right) = -\frac{\zeta[5/2]}{2\sqrt{2}\pi^{3/2}}\frac{k_B^{5/2}T^{5/2}m^{3/2}\ell}{h^3} + \frac{23\pi k_B T}{2016\,\ell^2}. \quad (37)$$

It is easily see that the first terms of (36) and (37) corresponding to bulk grand canonical energy, which times $(L_x \times L_y)/V$ reduce to grand canonical energy density (21), where V is volume of the system. The most importance are the second terms in bracket of (36) and (37), which produce the Casimir energy, those are positive energies

$$\Omega_{\mathrm{C}}^{(A,id)}[T,\ell] = \frac{1}{\pi\beta}\frac{2\pi}{\lambda_B^2}\frac{23\alpha^2}{2016\pi} = \frac{23\pi k_B T}{252\,\ell^2}, \quad (38)$$

$$\Omega_{\mathrm{C}}^{(Z,id)}[T,\ell] = \frac{1}{2\pi\beta}\frac{2\pi}{\lambda_B^2}\frac{23\alpha^2}{2016\pi} = \frac{23\pi k_B T}{2016\,\ell^2}. \quad (39)$$

From (38), (39) one finds the Casimir force acts on per unit area of the plate are repulsive force

$$F_{\mathrm{C}}^{(A,id)}[T,\ell] = -\frac{\partial\Omega_{\mathrm{C}}^{(A,\mathrm{id})}[T,\ell]}{\partial\ell} = \frac{23\pi k_B T}{126p3} \approx 0.573\,\frac{k_B T}{\ell^3}, \quad (40)$$

$$F_{\mathrm{C}}^{(A,id)}[T,\ell] = -\frac{\partial\Omega_{\mathrm{C}}^{(Z,\mathrm{id})}[T,\ell]}{\partial\ell} = \frac{23\pi k_B T}{1008\,\ell^3} \approx 0.0716\frac{k_B T}{p3} \quad (41)$$

Eqs.(40), (41) show that Casimir force with Zaremba BC is 1/8 times the Casimir force with anti-periodic BC, that is similar to the ratio of massless scalar field [8].

Next, the Casimir force in an ideal Bose gas at temperature $T < T_c^{(0)}$ for usual BCs is established. In this case, the wave vector is quantized as

$$\int\frac{d^3\vec{k}}{(2\pi)^3} \to \frac{1}{\ell}\sum_i\int_0^{\infty}\frac{dk_\perp k_\perp}{2\pi}, k^2 \to k_\perp^2 + k_i^2, k_i = i/L, \quad (42)$$

where [10]

$$\begin{cases} L = \ell/2\pi, & i = 0, \pm1, \pm2 \ldots \text{for periodic BC}, \\ L = \ell/\pi, & i = 0,1,2 \ldots \text{ for Neumann BC}, \\ L = \ell/\pi, & i = 1,2 \ldots \text{ for Dirichlet BC}. \end{cases}$$

By using (17), (19), and quantization conditions (42), the grand canonical energy per an unit area of plate defined as

$$\Omega^{(\mathrm{P,id})}[T,\ell]=-\frac{1}{2\pi\beta}\sum_{i=-\infty}^{\infty}\sum_{j=1}^{\infty}\frac{1}{j}\int_0^{\infty}d\,k_{\perp}k_{\perp}e^{-\frac{j}{4\pi}\left(\lambda_B^2k_{\perp}^2+\frac{\lambda_B^2i^2}{L^2}\right)},\qquad(43)$$

$$\Omega^{(\mathrm{N,id})}[T,p]=-\frac{1}{2\pi\beta}\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\frac{1}{j}\int_0^{\infty}d\,k_{\perp}k_{\perp}e^{-\frac{j}{4\pi}\left(\lambda_B^2k_{\perp}^2+\frac{\lambda_B^2i^2}{L^2}\right)},\qquad(44)$$

$$\Omega^{(\mathrm{D,id})}[T,\ell]=-\frac{1}{2\pi\beta}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{1}{j}\int_0^{\infty}d\,k_{\perp}k_{\perp}e^{-\frac{j}{4\pi}\left(\lambda_B^2k_{\perp}^2+\frac{\lambda_B^2i^2}{L^2}\right)},\qquad(45)$$

for periodic BC, Neumann BC, and Dirichlet BC, respectively. Combining (43) -(45) with (28), (30), and (31), we obtain

$$\Omega^{(\mathrm{P,id})}[T,\ell]=-\frac{1}{\pi\beta}\frac{2\pi}{\lambda_B^2}\left(\frac{\pi\zeta[5/2]}{\alpha}+\frac{23\alpha^2}{1512\pi}\right)=-\frac{\zeta[5/2]}{2\sqrt{2}\pi^{3/2}}\frac{k_B^{5/2}\tau^{5/2}m^{3/2}\ell}{\hbar^3}-\frac{23\pi k_BT}{189\ell^2},\qquad(46)$$

$$\Omega^{(\mathrm{N,id})}[T,\ell]=-\frac{1}{2\pi\beta}\frac{2\pi}{\lambda_B^2}\left(\frac{\pi\zeta[5/2]}{\alpha}+\frac{49}{60}+\frac{23\alpha^2}{1512\pi}\right)=-\frac{\zeta[5/2]}{2\sqrt{2}\pi^{3/2}}\frac{k_B^{5/2}T^{5/2}m^{3/2}\ell}{\hbar^3}-\frac{49}{120\pi}\frac{k_B^2T^2m}{h^2}-\frac{23\pi k_BT}{1512\ell^2},\qquad(47)$$

$$\Omega^{(\mathrm{D,id})}[T,\ell]=-\frac{1}{2\pi\beta}\frac{2\pi}{\lambda_B^2}\left(\frac{\pi\zeta[5/2]}{\alpha}-\frac{49}{60}+\frac{23\alpha^2}{1512\pi}\right)=-\frac{\zeta[5/2]}{2\sqrt{2}\pi^{3/2}}\frac{k_B^{5/2}T^{5/2}m^{3/2}\ell}{\hbar^3}+\frac{49}{120\pi}\frac{k_B^2T^2m}{h^2}-\frac{23\pi k_BT}{1512\ell^2}.\qquad(48)$$

It is easily to analyze that the first terms in Eqs.(46), (47), (48) define bulk energy. The second terms of Eqs.(47), (48) are surface energy, which is canceled out in (46). The Casimir-type energies are identified by the last terms of them as

$$\Omega_C^{(\mathrm{P,id})}[T,\ell]=-\frac{23\pi k_BT}{189\ell^2}\approx-0.3823\frac{k_BT}{p2},\qquad(49)$$

$$\Omega_C^{(\mathrm{N,id})}[T,P]=\Omega_C^{(\mathrm{D,id})}[T,\ell]=-\frac{23\pi k_BT}{1512\ell^2}\approx-0.04778\frac{k_BT}{\ell^2}.\qquad(50)$$

The Casimir-type forces identified through $-\partial_p\Omega_C[T,l]$:

$$F_C^{(\mathrm{P,id})}[T,P]=-\frac{\partial\Omega_C^{(\mathrm{P,id})}[T,\ell]}{\partial\ell}=-\frac{46\pi k_BT}{189\ell^3},\qquad(51)$$

$$F_C^{(\mathrm{N,id})}[T,\ell]=F_C^{(\mathrm{D,id})}[T,P]=-\frac{\partial\Omega_C^{(\mathrm{N,id})}[T,\ell 1}{\partial\ell}=-\frac{23\pi k_BT}{756\ell^3}\qquad(52)$$

DISCUSSIONS

By invoking statistic mechanic formalism with the helps of Poison and Euler-MacLaurin summations, the authors of Ref.[10] proved that the Casimir-type energy is defined as $\zeta[3]k_BT/\pi\ell^2\approx-0.3826k_BT/\ell^2$ for periodic BC and $\zeta[3]/k_BT/8\pi\ell^2\approx-0.04782k_BT/\ell^2$ for Neumann and Dirichlet BCs. That is exactly coincides with our results show in Eqs.(49), (50). Using the same way, the authors of Refs.[14], [15] found $F_C^{(A,id)}[T,\ell\ ]=3\zeta[3]k_BT/2\pi\ell^3\approx 0.574k_BT/\ell^3, F_C^{(Z,id)}[T,\ell]=3\zeta[3]k_BT/16\pi\ell^3\approx0.0717k_BT/\ell^3$,for the Casimir-type force, which was recovered in Eqs.(40), (41). In Refs. [10, 14,15], the Casimir-type energy as well as the Casimir-type force were obtained by combining Poison summation and Euler-MacLaurin summation for two regions small and lager i, ℓ. In which the results for usual BCs (periodic, Neumann, Dirichlet) and special BCs (anti-periodic, Zaremba) were established separately. In our calculations, with the help of Taylor expansion, we only use Euler-MacLaurin formula for any range of i, ℓ. Moreover, not only the Casimir-type energy and the Casimir-type force but also the surface tension for any BCs were produced from Eqs.(33) and (34).

CONCLUSIONS

In the previous sections, we have been established the thermodynamical grand canonical energy of a weakly interacting Bose gas, which is one of the basic theories to studying of Casimir-type effect of Bose gas in one loop approximation in both zero temperature and finite temperature regimes. The absence of chemical potential in grand canonical potential implying that one-loop approximation of quantum field theory only suitable for area below critical temperature. Which is the reason why in this paper the effect in the region above critical temperature has been not considered.

In the three dimension space compacted along Oz-direction, by a simpler way than the one before, we established explicit formulae of the negative Casimir-type energy which arise to the attractive Casimir-type forces for usual BCs, and the positive Casimir-type energy which arise to the repulsive Casimir-type forces for special BCs. In which the Casimir-type energy is proportional to ℓ^{-2}, which leads to the Casimir-type force decay as ℓ^{-3}. Beside, the exactly surface tension for any BCs were defined. The calculation method has been mentioned in this studying as well as the results produced from it will be the important fundamentals for our studying of the weakly interacting Bose gas in the future.

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LỰC CASIMIR-TYPE CỦA NGƯNG TỤ BOSE-EINSTEIN LÝ TƯỞNG TRONG PHA ĐỐI XỨNG BỊ PHÁ VỠ

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Tóm tắt: *Chúng tôi nghiên cứu hiệu ứng Casimir-type trong hệ ngưng tụ Bose-Einstein không tương tác bị giới hạn giữa hai bản song song trong mặt phẳng (x,y) và cách nhau một khoảng* ℓ *dọc theo trục z với mọi điều kiện biên. Trong đó, năng lượng Casimir-type tỷ lệ với* ℓ^{-2} *dẫn tới hệ quả lực Casimir-type giảm khi khoảng cách giữa hai bản tăng theo quy luật* ℓ^{-3}.

Từ khóa: *Bose gas; Casimir force; Finite-size effect.*

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