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ON THE CASIMIR EFFECT IN A WEAKLY INTERACTING BOSE-EINSTEIN CONDENSATE GAS AT ZERO TEMPERATURE

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Abstract: *Within framework of the Bogoliubov theory of a dilute Bose gas, we investigate quantum fluctuation energy of one component weakly interacting Bose-Einstein condensate (BEC) gas confined between two parallel plates at zero temperature in grand canonical ensemble (GCE). The Casimir forces corresponding to Neumann, Dirichlet, Robin and periodic boundary conditions (BCs) of the k_Z-wave vector component are compared to the one obtained by quantum field theory (QFT).*

Keywords: *weakly interacting Bose gas; the Casimir effect; Finite-size effect.*

I. INTRODUCTION

Based on the Casimir's original calculation method [1], D.C. Robert and Y. Pomeau studied of Casimir effect in a dilute homogeneous BEC gas, that is restricted between two yery large parallel plates separated by a distance ℓ[2,3]. In which, Dirichlet BC is imposed to the k_z-wave vector component, and the Casimir energy is defined by subtracting excitation energy corresponds to continuous momentum spectrum from excitation energy corresponds to discrete momentum spectrum. This studying found out the well-known relation of Casimir pressure is $p_C = -\pi^2 h v_s/480\ell^4$, where ħ is the Planck's constant, v_s is the speed of sound in the system. **By this way, the authors of Ref.[4] proved that $\boldsymbol{p_C = -\pi^2 h v_s/30\ell^4}$ for periodic BC. These authors also developed multidimensional cut-off technique to solve Casimir effect problem in massless scalar field [5].** Later, the result in Ref.[4] was recovered within QFT framework in one-loop approximation, and series expansion of excitation energy about small value of ξ/ℓ[6], where ξ being healing length of condensate [7]. In Ref.[8], by applying the Hamiltonian formalism, the results in Refs.[2, 3, 4] once again confirmed, moreover higher terms of the Casimir pressure were calculated by zeta-functional regularization method. By using QFT combine cut-off momentum, N.V.Thu [9] obtained the result exactly coincides with the ones given in Refs.[3, 4]. Recently, we not only obtained a similar result [10] but also developed the calculating to the double-bubble approximation within framework of Cornwall-Jackiw-Tomboulis (CJT) effective potential in improyed Hartree−Fock approximation (IHF) [11]. In one-loop approximation, Casimir effect in two components Bose-Einstein condensate gas was first attained by N.V. Thu *et al* in Ref.[12].

The aim of this paper is to studying of the Casimir effect in a weakly interacting Bose-Einstein condensate gas at condensate phase based on Bogoliubov theory, and comparing the results with the ones were defined by using QFT before. The paper is organized as follows. In section 2, quantum fluctuation energy of a weakly interacting Bose gas is investigated in symmetry broken phase. The Casimir effect is considered in section 3. Conclusions are given in section 4 to close the paper.

II. THE QUANTUM FLUCTUATION ENERGY OF WEAKLY INTERACTING BOSE GAS

In this section we consider the quantum fluctuation energy of a weakly interacting Bose gas in infinite three-dimension space. According to the Bogoliubov theory [7], quantum fluctuation energy of a dilute boson gas at zero-temperature reads as

$$\mathcal{E}^q(\mathrm{k}) = \frac{1}{2}\sum_{\mathrm{k}\neq 0}\left[\varepsilon(\mathrm{k}) - gn_0 - \frac{\hbar^2\mathrm{k}^2}{2m} + \frac{m(gn_0)^2}{\hbar^2\mathrm{k}^2}\right]. \quad (1)$$

In which, k being the module of the wave vector $\vec{k}$, $g = 4\pi\hbar^2 a_s m^{-1} > 0$ determines the strength of repulsive interaction between two intraspecies atoms, a_s being s-wave scattering length, m is atomic mass, n_0 is identical atoms density,

$$\varepsilon(\mathrm{k}) = \sqrt{\frac{\hbar^2\mathrm{k}^2}{2m}\left(\frac{\hbar^2\mathrm{k}^2}{2m} + 2gn_0\right)} \quad (2)$$

defines elementary excitation in the system, that called Bogoliubov dispersion relation [10, 12]. Recently, Eq.(2) also found out by different methods. For examples, within perturbatiye framework of QFT [13], CJT formalism [14,11]. In dilute (weakly interacting) Bose gas, the diluteness condition is required, this means that $n_0 a_s^3 \ll 1$[13].

Assume that the system in the box $V = L_x \times L_y \times L_z$, from now on one notes $\ell \equiv L_z$. For simplicity, the wave vector is assumed satisfies Neumann BC has the form [15]

$$\mathrm{k} = \sqrt{k_x^2 + k_y^2 + k_z^2}, \quad (3)$$

$$k_x = \frac{\pi n_x}{L_x}, k_y = \frac{\pi n_y}{L_y}, k_y = \frac{\pi n_z}{\ell},$$

$$n_x, n_y, n_z = 0, 1, 2. \quad (4)$$

Plugging (2), (3) and (4) into (1) yields

$$\mathcal{E}^q(\mathrm{k}) = \frac{1}{2}\sum_{n_x=0}^{\infty}\sum_{n_y=0}^{\infty}\sum_{n_z=0}^{\infty}[\sqrt{\frac{\hbar^2\left(\frac{\pi^2 n_x^2}{L_x^2} + \frac{\pi^2 n_y^2}{L_y^2} + \frac{\pi^2 n_z^2}{\ell^2}\right)}{2m}\left(\frac{\hbar^2\left(\frac{\pi^2 n_x^2}{L_x^2} + \frac{\pi^2 n_y^2}{L_y^2} + \frac{\pi^2 n_z^2}{\ell^2}\right)}{2m} + 2gn_0\right)}$$

$$-gn_0 - \frac{\hbar^2\left(\frac{\pi^2 n_x^2}{L_x^2} + \frac{\pi^2 n_y^2}{L_y^2} + \frac{\pi^2 n_z^2}{\ell^2}\right)}{2m} + \frac{m(gn_0)^2}{\hbar^2\left(\frac{\pi^2 n_x^2}{L_x^2} + \frac{\pi^2 n_y^2}{L_y^2} + \frac{\pi^2 n_z^2}{\ell^2}\right)}]\ . \quad (5)$$

In infinite space, namely $(L_x, L_y, \ell) \to \infty$, triple sums in Eq.(5) can be replaced by triple integrals. Eq.(5) is rewritten as

$$\mathcal{E}^q(\mathrm{k}) = \frac{1}{2}\int_0^\infty d\,n_x \int_0^\infty d\,n_y \int_0^\infty d\,n_z [\sqrt{\frac{\hbar^2\left(\frac{\pi^2 n_x^2}{L_x^2}+\frac{\pi^2 n_y^2}{L_y^2}+\frac{\pi^2 n_z^2}{\ell^2}\right)}{2m}\left(\frac{\hbar^2\left(\frac{\pi^2 n_x^2}{L_x^2}+\frac{\pi^2 n_y^2}{L_y^2}+\frac{\pi^2 n_z^2}{\ell^2}\right)}{2m}+2gn_0\right)}$$

$$-gn_0 - \frac{\hbar^2\left(\frac{\pi^2 n_x^2}{L_x^2}+\frac{\pi^2 n_y^2}{L_y^2}+\frac{\pi^2 n_z^2}{\ell^2}\right)}{2m} + \frac{m(gn_0)^2}{\hbar^2\left(\frac{\pi^2 n_x^2}{L_x^2}+\frac{\pi^2 n_y^2}{L_y^2}+\frac{\pi^2 n_z^2}{\ell^2}\right)}] \,. \tag{6}$$

Apply changing of variable method by setting: $x = \frac{\pi n_x}{L_x}, y = \frac{\pi n_y}{L_y}, z = \frac{\pi n_z}{\ell}$, one has

$$\mathcal{E}^q(\mathrm{k}) = \frac{1}{2}\frac{L_x}{\pi}\frac{L_y}{\pi}\frac{\ell}{\pi}\int_0^\infty d\,x \int_0^\infty d\,y \int_0^\infty d\,z [\sqrt{\frac{\hbar^2(x^2+y^2+z^2)}{2m}\left(\frac{\hbar^2(x^2+y^2+z^2)}{2m}+2gn_0\right)}$$

$$-gn_0 - \frac{\hbar^2(x^2+y^2+z^2)}{2m} + \frac{m(gn_0)^2}{\hbar^2(x^2+y^2+z^2)}] \,. \tag{7}$$

In spherical coordinate, Eq.(7) has the form

$$\mathcal{E}^q(\mathrm{k}) = \frac{1}{2}\frac{L_x}{\pi}\frac{L_y}{\pi}\frac{\ell}{\pi}\frac{\pi}{2}\int_0^\infty d\,r r^2 \left[\sqrt{\frac{\hbar^2 r^2}{2m}\left(\frac{\hbar^2 r^2}{2m}+2gn_0\right)} - gn_0 - \frac{\hbar^2 r^2}{2m} + \frac{m(gn_0)^2}{\hbar^2 r^2}\right], \tag{8}$$

where $r^2 = x^2 + y^2 + z^2$.

Perform above integral over $r \in [0, \Lambda]$, and takes limit $\Lambda \to \infty$ one finds

$$\mathcal{E}^q(\mathrm{k}) = \frac{1}{2} g n_0^2 V \frac{128}{15\sqrt{\pi}}\sqrt{n_0 a_s^3} = \frac{1}{2} g n_0 N \frac{128}{15\sqrt{\pi}}\sqrt{n_0 a_s^3}, \tag{9}$$

II. THE CASIMIR EFFECT

In order to deal with the Casimir effect, we assume that the system is confined to two parallel plates of area $L_x \times L_y$, they are perpendicular to Oz-axis and separated by a distance ℓ. In the following, one only discusses finite-size effect along z-direction, this means that $\ell^2 \ll L_x \times L_y$. Furthermore, distance between plates has to be enough for manifestation of quantum fluctuation [4], namely $\ell/\xi \gg 1$, where $\xi = h/\sqrt{2mgn_0}$. Note that the system is considered in GCE, this means that it is connected with a bulk reservoir of condensation with condensate density equal to n_0.

Owing to compactification of z -direction, Eq.(5) become

$$\mathcal{E}^q(\mathrm{k}) = \frac{1}{2}\frac{L_x}{\pi}\frac{L_y}{\pi}\frac{\pi}{2}\int_0^\infty d\,x \int_0^\infty d\,y \sum_{n_z=0}^{\infty} [\sqrt{\frac{\hbar^2(x^2+y^2+\pi^2 n_z^2/\ell^2)}{2m}\left(\frac{\hbar^2(x^2+y^2+\pi^2 n_z^2/\ell^2)}{2m}+2gn_0\right)}$$

$$-gn_0 - \frac{\hbar^2(x^2+y^2+\pi^2 n_z^2/\ell^2)}{2m} + \frac{m(gn_0)^2}{\hbar^2(x^2+y^2+\pi^2 n_z^2/\ell^2)}]. \tag{10}$$

In polar coordinate, Eq.(10) reads as

$$\mathcal{E}^q(\mathrm{k}) = \frac{1}{2}\frac{L_x}{\pi}\frac{L_y}{\pi}\frac{\pi}{2}\sum_{n_z=0}^\infty \int_0^\infty drr\ [\sqrt{\frac{\hbar^2(r^2+\pi^2 n_z^2/p)}{2m}\left(\frac{\hbar^2(r^2+\pi^2 n_z^2/\ell^2)}{2m}+2gn_0\right)}$$

$$-gn_0 - \frac{\hbar^2(r^2+\pi^2 n_z^2/\ell^2)}{2m} + \frac{m(gn_0)^2}{\hbar^2(r^2+\pi^2 n_z^2/\ell^2)}], \tag{11}$$

in which $r^2 = x^2 + y^2$.

Integrating over $r \in [0, \Lambda]$ of (11), and take limit $\Lambda \to \infty$ yields

$$\mathcal{E}^q(\mathrm{k}) = \frac{1}{2}\frac{L_x}{\pi}\frac{L_y}{\pi}\frac{\pi}{2}\sum_{n_z=0}^{\infty}\Big[\frac{\pi^2 g n_0 n_z^2}{2\ell^2} - \frac{\pi g n_0 n_z}{4\ell}\sqrt{\frac{\pi^2 n_z^2}{\ell^2} + \frac{4mgn_0}{h^2}} + \frac{\pi^4 h^2 n_z^4}{8m\ell^4}$$

$$-\frac{\pi^3 \hbar^2 n_z^3}{8m\ell^3}\sqrt{\frac{\pi^2 n_z^2}{\ell^2} + \frac{4mgn_0}{h^2}} + \frac{mg^2 n_0^2}{h^2}\left(\frac{1}{4} + \ln\left[\frac{1}{2} + \frac{\ell}{2\pi n_z}\sqrt{\frac{\pi^2 n_z^2}{\ell^2} + \frac{4mgn_0}{h^2}}\right]\right)\Big]. \quad (12)$$

By employing Euler-Maclaurin formula in the form [17]

$$\sum_{n=0}^{\infty} f[n] = \int_0^{\infty} f[x]dx + \frac{1}{2}(f[0] + f[\infty]) + \frac{1}{12}\left(f^{(1)}[\infty] - f^{(1)}[0]\right)$$

$$-\frac{1}{720}\left(f^{(3)}[\infty] - f^{(3)}[0]\right) + \frac{1}{30240}\left(f^{(5)}[\infty] - f^{(5)}[0]\right] \ldots \quad (13)$$

with $f[n_z] \equiv \mathcal{E}^q(\mathrm{k})$, Eq. (12) produces

$$\mathcal{E}^q(\mathrm{k}) = (L_x \times L_y \times \ell)\frac{1}{2}gn_0^2\frac{128}{15\sqrt{\pi}}\sqrt{n_0 a_s^3} + (L_x \times L_y)\frac{\sqrt{\pi}h\sqrt{g}n_0^{3/2}}{4\sqrt{m}}\sqrt{n_0 a_s^3} - (L_x \times L_y)\frac{\pi^2 h\sqrt{gn_0}}{480\sqrt{m}\ell^3}. \quad (14)$$

Recast that the volume of the system is $V = L_x \times L_y \times \ell$, and $v_s = \sqrt{gn_0}/m$ is speed of sound. Using Eq.(14) one can defined quantum fluctuation energy per an unit area of plates is

$$E_q^{(N)}(\mathrm{k}) = \frac{1}{2}gn_0^2\ell\frac{128}{15\sqrt{\pi}}\sqrt{n_0 a_s^3} + \frac{\sqrt{\pi}hn_0 v_s}{4}\sqrt{n_0 a_s^3} - \frac{\pi^2 h v_s}{480\ell^3}. \quad (15)$$

In the cases either periodic BC, Dirichlet BC, or Robin BC is imposed, the k_z-component of the wave vector is quantized as following [9, 15]

$$k_z = n_z/L \quad (16)$$

with $L = \ell/2\pi, n_z = 0, \pm 1, \pm 2 \ldots$ for periodic BC, $L = \ell/\pi$, $n_z = 1,2 \ldots$ for Dirichlet BC, $L = \left(\ell + \xi/\sqrt{2}\right)/\pi$, $n_z = 1,2 \ldots$ for Robin BC.

Based on the Eq.(15) and quatizion condition (16), the quantum fluctuation energy per an unit area of plates correspond to periodic BC, Dirichlet BC, and Robin BC, respectively, are defined as

$$E_q^{(P)}(\mathrm{k}) = \frac{1}{2}gn_0^2\ell\frac{128}{15\sqrt{\pi}}\sqrt{n_0 a_s^3} - \frac{\pi^2 h v_s}{30\ell^3}, \quad (17)$$

$$E_q^{(D)}(\mathrm{k}) = \frac{1}{2}gn_0^2\ell\frac{128}{15\sqrt{\pi}}\sqrt{n_0 a_s^3} - \frac{\sqrt{\pi}hn_0 v_s}{4}\sqrt{n_0 a_s^3} - \frac{\pi^2 h v_s}{480\ell^3}, \quad (18)$$

$$E_q^{(R)}(\mathrm{k}) = \frac{1}{2}gn_0^2\left(\ell + \xi/\sqrt{2}\right)\frac{128}{15\sqrt{\pi}}\sqrt{n_0 a_s^3} - \frac{\sqrt{\pi}\hbar n_0 v_s}{4}\sqrt{n_0 a_s^3} - \frac{\pi^2 h v_s}{480\left(\ell + \xi/\sqrt{2}\right)^3}. \quad (19)$$

In the case the two walls do not exist, which leads to quantization condition of the waye yector is canceled out, thus Eqs.(15), (17), (18), (19) become the same each other $E_q^{(N)}(\mathrm{k}) = E_q^{(P)}(\mathrm{k}) = E_q^{(D)}(\mathrm{k}) = E_q^{(R)}(\mathrm{k}) = \frac{1}{2}gn_0^2\ell\frac{128}{15\sqrt{\pi}}\sqrt{n_0 a_s^3}$, which times the size of the plates to excitation energy density of bulk condensate reservoir as defined in Eq.(9). This implies that the first terms in the right hand side of Eqs.(15), (17), (18), (19) define "bulk" energy, wheres the last terms decay as ternary power of distance between the plates, which manifestation of Casimir energy

$$E_C^{(R)}(\mathrm{k}) = -\frac{\pi^2 h v_s}{30\ell^3}. \quad (20)$$

$$E_C^{(N)}(\mathrm{k}) = E_C^{(D)}(\mathrm{k}) = -\frac{\pi^2 \hbar v_s}{480\ell^3}, \quad (21)$$

$$E_C^{(R)}(\mathrm{k}) = -\frac{\pi^2 h v_s}{480(\ell+\xi/\sqrt{2})^3}. \quad (22)$$

Beside the second terms of Eqs.(15), (18), (19) in proportion to area of plates thus they are surface energy, which is annihilated for periodic BC as in Eq.(17).

The Casimir forces act on per an unit of the plates produce from the Casimir energy through $-\partial_\ell E_C(\mathrm{k})$ as following

$$F_C^{(P)}(\ell) = -\frac{\pi^2 h v_s}{10\ell^4}, \quad (23)$$

$$F_C^{(N)}(\ell) = F_C^{(D)}(\ell) = -\frac{\pi^2 h v_s}{160\ell^4}, \quad (24)$$

$$F_C^{(R)}(\ell) = -\frac{\pi^2 h v_s}{160(\ell+\xi/\sqrt{2})^4}, \quad (25)$$

which are the attractive forces arsing from the negative energies.

In order to compare our results to the ones in Refs.[4, 6, 3, 9], we scale Eqs.(23), (24), (25) by $F_0 = h v_s/\xi^4$ as following

$$F_C^{(P)}(\ell) = -F_0 \frac{\pi^2}{10}\frac{1}{L^4}, \quad (26)$$

$$F_C^{(N)}(\ell) = F_C^{(D)}(\ell) = -F_0 \frac{\pi^2}{160}\frac{1}{L^4}, \quad (27)$$

$$F_C^{(R)}(\ell) = -F_0 \frac{\pi^2}{160}\frac{1}{(L+1/\sqrt{2})^4}. \quad (28)$$

where $L = \ell/\xi$ is the distance between two plates in dimensionless form. By this way, the results of Refs.[4, 6, 3, 9] read as

$$F_C^{(P)}(\ell) = -F_0 \frac{\pi^2}{30}\frac{1}{L^4}, \quad (29)$$

$$F_C^{(D)}(\ell) = -F_0 \frac{\pi^2}{480}\frac{1}{L^4}, \quad (30)$$

$$F_C^{(R)}(\ell) = -F_0 \frac{\pi^2}{480}\frac{1}{(L+1/\sqrt{2})^4}. \quad (31)$$

Eqs.(26), (27), (28) and Eqs.(29), (30), (31) show that results obtained by Bogoluibov theory are three times bigger than the ones obtained by QFT. Based on these equations we plot Fig.1, which represent the evolution of the Casimir forces over distance between two plates with different BCs, in which the solid lines show our results, the dashed lines show the results were obtained within QFT. Easily recognize that the force corresponds to periodic BC much greater than the ones corresponds to other BCs, the Casimir force is minimum when Robin BC is imposed.

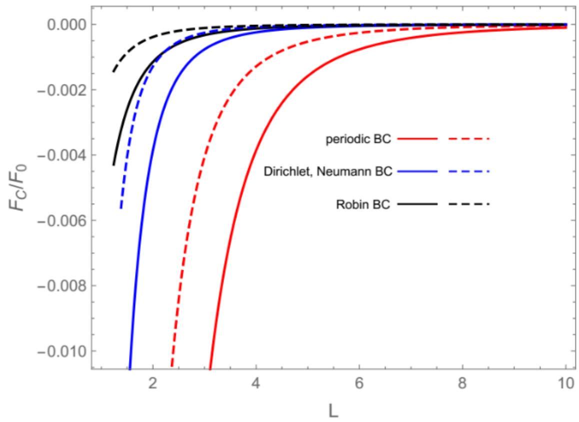

Figure 1. *(Color online) The influence of the distance between two plates on the Casimir forces. The solid lines show the results obtained by the Bogoliubov theory. The dashed lines show the results obtained by the QFT.*

IV. DISCUSSIONS AND CONCLUSIONS

In the foregoing sections, the quantum fluctuation energy and the Casimir effect of a weakly interacting BEC are studied within the Bogoliuboy theory for any usual BCs. The main results are:

(i) Exactly recovered the well-known quantum fluctuation energy formula of the system in infinite space, which was reported in previous researches [7, 16].

(ii) With the help of Euler-Maclaurin formula, the Casimir energy as well as Casimir force corresponding to any usual BCs are found. The Casimir forces are attractive forces, proportional to the speed of sound, and decay as fourth power of the distance between two plates. The minimum of the forces appears when Robin BC is imposed. These laws the same as the one were defined based on the QFT.

(iii) We not only investigate the Casimir effect but also clearly define the surface energy of the system, which is a positive quantity for Neumann BC, negative for Dirichlet and Robin BCs, canceled out for periodic BC.

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REFERENCES

[1] Casimir H. B. G., 1948. Proc. K. Ned . Akad. Wet. 51, 793.

[2] Roberts D. C., Pomeau Y., 2008 arXiv:cond -mat/0503757.

[3] Roberts D. C., Pomeau Y., 2005 Casimir-Like Force Arising from Quantum Fluctuations in a Slowly Moving Dilute Bose-Einstein Condensate. Phys. Rev. Lett. 95, 145303.

[4] Edery A., 2006. Casimir forces in Bose-Einstein condensates: Finite size effects in three-dimensional rectangular cavities. J. Stat. Mech. P06007.

[5] Edery A., 2006. Multidimensional cut-off technique, odd-dimensional Epstein zeta functions and Casimir energy of massless scalar fields. J. Phys. A: Math. Gen. 39, 685-712.

[6] Schiefele J. and Henkel C., 2009 Casimir energy of a BEC: from moderate interactions to the ideal gas J. Phys. A: Math. Theor. 42, 045401.

[7] Pitaevskii L. , Stringari S., 2003. Bose-Einstein Condensation Oxford University Press.

[8] Biswas S. *et. al.*, 2010. Casimir force on an interacting Bose-Einstein condensate J. Phys. B43, 085305.

[9] Thu N. V., 2018. The forces on a single interacting Bose-Einstein condensate Phys. Lett. A 382,1074.

[10] Pham The Song, Nguyen Van Thu, 2021. The Casimir Effect in a Weakly lnteracting Bose Gas. J Low Temp Phys 202, 160−174.

[11] Thu N. V., Song P. T., 2020 .Casimir effect in a weakly interacting Bose gas confined by a parallel plate geometry in improved Hartree-Fock approximation Physica A 540, 123018.

[12] Thu N. V., Theu L. T., 2017. Casimir Force of Two-Component Bose-Einstein Condensates Confined by a Parallel Plate Geometry. J. Stat. Phys. 1681-10.

[13] Andersen J. O., 2004. Theory of the weakly interacting Bose gas. Rev. Modern Phys. 76, 599.

[14] Phat T. H., Hoa L. V., Anh N. T., Long N. V., 2009. Bose-Einstein condensation in binary mixture of Bose gases. Ann. Phys. 324, 2074.

[15] Martin P. A. and Zagrebnov V. A., 2006. The Casimir effect for the Bose-gas in slabs. Europhys. Lett. 73, 15-20.

[16] Lee T. D. and Yang C. N., 1957. Many-Body Problem in Quantum Mechanics and

Quantum Statistical Mechanics. Phys. Rev. 105, 1119.

[17] Arfken G.B. , Weber H.J., 2005. Mathematical Methods for Physicists ElsevierAcademic Press.

HIỆU ỨNG CASIMIR TRONG KHÍ NGƯNG TỤ BOSE-EINSTEIN TƯƠNG TÁC YẾU Ở NHIỆT ĐỘ KHÔNG

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Tóm tắt: *Trong khuôn khổ của lý thuyết Bogoliubov, chúng tôi nghiên cứu năng lượng thăng giáng lượng tử của khí ngưng tụ Bose-Einstein một thành phần tương tác yếu bị giới hạn bởi hai bản phẳng song song ở nhiệt độ không trong thống kê chính tắc lớn. Lực Casimir tương ứng với các điều kiện biên Neumann, Dirichlet, Robin và tuần hoàn của thành phần véc-tơ sóng* k_z *được so sánh với kết quả thu được từ lý thuyết trường lượng tử.*

Từ khóa: *khí Bose tương tác yếu; hiệu ứng Casimir; Hiệu ứng kích thước hữu hạn.*

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